Branching pomsets and event structures
(oral communication)

Luc Edixhoven$^{1,2}$
José Proença$^3$
Sung-Shik Jongmans$^{1,2}$
Ilaria Castellani$^4$

$^1$Open University of the Netherlands  $^2$CWI
$^3$CISTER, ISEP, Polytechnic Institute of Porto  $^4$INRIA, Université Côte d’Azur

ICE 2023
Branching pomsets for choreographies

Luc Edixhoven\textsuperscript{1,2} \quad Guillermina Cledou\textsuperscript{3,4}

Sung-Shik Jongmans\textsuperscript{1,2} \quad José Proença\textsuperscript{5}

\textsuperscript{1} Open University of the Netherlands \quad \textsuperscript{2} CWI

\textsuperscript{3} HASLab, INESC TEC \quad \textsuperscript{4} University of Minho

\textsuperscript{5} CISTER, ISEP, Polytechnic Institute of Porto

ICE 2022
Some context

Branching pomsets for choreographies

\[ [a \rightarrow b:x ; ((b \rightarrow c:x + d \rightarrow e:x) \ || \ c \rightarrow a:x)] \]
Conclusions and future work

Summary

• Branching pomsets
• Compact for both concurrency and choice
• Can express the same behaviour as choreographies

Future work

• Framework improvements: \( n \)-ary choices, partial order, loops
• Static analysis: realisability

https://lmf.di.uminho.pt/b-pomset/
“What about event structures?”
Branching pomsets and event structures
(oral communication)

Luc Edixhoven$^{1,2}$
José Proença$^3$
Sung-Shik Jongmans$^{1,2}$
Ilaria Castellani$^4$

$^1$Open University of the Netherlands   $^2$CWI
$^3$CISTER, ISEP, Polytechnic Institute of Porto   $^4$INRIA, Université Côte d'Azur

ICE 2023
Outline

• **Branching pomsets**: a generic model for concurrency
• **Event structures**: a brief overview of the landscape
• **Comparison**: relative expressiveness
Branching pomsets and event structures

**Basis:** partially ordered multisets / pomsets (Pratt 1986)

- a set of events
  - above: \{a, b, c, d, e, f, g, h\}
- a partial order on the events
  - above: the reflexive and transitive closure of the arrows
- a labelling function from events to some set of labels
  - above: omitted / identity (irrelevant for this talk)
Extension: choices

- expressing choices with pomsets requires a set of pomsets
- with many choices, this set may become exponentially large
- solution: add a representation of choices
Branching pomsets

**Choice model**: branching structure

- add branching structure; a tree whose leaves are the events above: \( \{a, b, g, h, C_1, C_2\} \), where \( C_1 = \{\{c\}, \{d\}\} \) and \( C_2 = \{\{e\}, \{f\}\} \)

- replace the partial order with a precedence relation, whose reflexive and transitive closure is a partial order above: the arrows
Branching pomsets

For comparison: the corresponding set of pomsets

\[
\begin{array}{c}
  a \rightarrow c \rightarrow e \rightarrow g \\
  b \downarrow \downarrow \downarrow \\
  h
\end{array}
\quad
\begin{array}{c}
  a \\
  b \downarrow \downarrow \downarrow \\
  h
\end{array}
\]

\[
\begin{array}{c}
  a \rightarrow c \\
  b \downarrow \downarrow \\
  f \rightarrow h
\end{array}
\quad
\begin{array}{c}
  a \rightarrow g \\
  b \downarrow \downarrow \\
  f \rightarrow h
\end{array}
\]

\[
\begin{array}{c}
  a \rightarrow e \rightarrow g \\
  b \downarrow \downarrow \downarrow \\
  h
\end{array}
\quad
\begin{array}{c}
  a \\
  b \downarrow \downarrow \downarrow \\
  d \rightarrow f \rightarrow h
\end{array}
\]

\[
\begin{array}{c}
  a \rightarrow g \\
  b \downarrow \downarrow \\
  f \rightarrow h
\end{array}
\quad
\begin{array}{c}
  a \\
  b \downarrow \downarrow \\
  d \rightarrow f \rightarrow h
\end{array}
\]
**Semantics:** refining $\Rightarrow$ resolving any number of choices
**Semantics**: enabling (followed by firing) $\Rightarrow$ refining s.t. the chosen event is minimal and top-level, resolving no more than necessary
Semantics: enabling (followed by firing) ⇒ refining s.t. the chosen event is minimal and top-level, resolving no more than necessary
Also: nested choices
**Choice model:** conflict relation

- add conflict relation; two conflicting events may not occur together in the same execution

above: \{ (c, d), (e, f) \}

- most classes of event structures define variations on causality and/or conflicts
Event structures

**Landscape** (partial): static and dynamic classes of event structures

Prime → Asymmetric → Growing

Bundle → Extended Bundle

Flow → Stable → Dual ← Shrinking

Dynamic Causality → Resolvable Conflict → HDES

Arrows represent (strict) inclusion in terms of expressiveness

Figure: Arbach, Karcher, Peters and Nestmann, Dynamic causality in event structures (2018)
Event structures

Landscape (partial): static and dynamic classes of event structures

Prime → Asymmetric → Growing

Bundle → Extended Bundle

Flow → Stable → Dual → Shrinking

Dynamic Causality
Resolvable Conflict

HDES

Arrows represent (strict) inclusion in terms of expressiveness

Figure: Arbach, Karcher, Peters and Nestmann, Dynamic causality in event structures (2018)

Most relevant for this talk: growing and shrinking causality ⇒ dynamically adding and removing causalities
Comparison

Prime → Growing

Bundle → Extended Bundle

→ Shrinking

→ Dynamic Causality

→ Resolvable Conflict
Dynamic causality with counters: replaced dynamic causality event structures with a new variant with nice property; the order of events is irrelevant for the resulting causal state

As a result: uniformly defined semantics for all shown classes
Generic proof: inclusion in event structures for resolvable conflict of any class of event structures where the causal state is order-independent, including dynamic counters.
Next up: branching pomsets
**Non-inclusion**: not all prime event structures expressible as branching pomsets — would need overlapping boxes

```
\[ \begin{array}{ccc}
  & a & c \\
  & \# & \# \\
  b & \# & d \\
\end{array} \] ```
**Non-inclusion**: not all branching pomsets expressible as growing causality event structures — would need disjunctive causality
Comparison

Branching Pomsets

Prime → Growing

Bundle → Extended Bundle

Shrinking

Dynamic Counters

Resolvable Conflict

Non-inclusion: not all branching pomsets expressible as extended bundle event structures — $c$ can be disabled and then re-enabled
Non-inclusion: not all branching pomsets expressible as shrinking causality event structures — $c$ can be disabled and then re-enabled
Consequently: branching pomsets incomparable with prime, growing causality, extended bundle and shrinking causality event structures
Comparison

**Inclusion**: subset of branching pomsets, dubbed *tree-like*, can be expressed as prime event structures.
**Inclusion**: same generic proof as for event structures also holds for branching pomsets; they can all be expressed as event structures for resolvable conflict.
Inclusion conjecture: dynamic causality event structures (with counters) may be powerful enough to express all branching pomsets; no proof yet
Conclusions and future work

Summary

• branching pomsets as a generic model for concurrency
• comparison with various classes of event structures
• interesting behaviour: incomparable with most, included in some more expressive classes of dynamic event structures

Future work

• proving or disproving the dynamic counters conjecture
• study the expressiveness of branching pomsets with overlapping boxes
• expand static analysis of branching pomsets