Catastrophe by Design

Destabilizing Wasteful Technologies & The Phase Transition from Proof of Work to Proof of Stake

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Energy Consumption of PoW

Proof of Work (PoW): Security $\iff$ Work $\iff$ Energy Consumption

- 1 BTC transaction = 775.818 VISA transactions.
- BTC consumes more energy than Finland and Pakistan.
- Energy consumption doubles every year.
- BTC is only one out of many PoW blockchains, e.g., Ethereum.

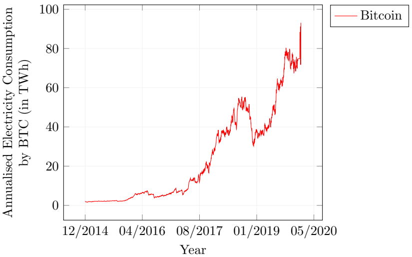


Figure: The Cambridge Bitcoin Electricity Consumption Index (CEBCI)

Transtition to PoS

Proof of Stake (PoS) has equivalent provable guarantees to PoW. But:

- More work implies more safety – more reliable applications (e.g., BTC).
- When all PoW it is individually rational to also PoW.
- Even worse PoW is evolutionary stable: small groups of adopters of alternative technologies are doomed to fail.

These observations hint towards a game-theoretic model.

Model I: Agents and Strategies

A population p of agents, investors or miners (physical or virtual)

- Mass $K > 0$: total available capital or resources, e.g., money, hardware or electricity.
- Strategies: two available technologies, W (costly), and S.
- Investment cost: $\gamma > 0$ for W and 0 for S.
- Population states: $X = \{(x, 1 - x) : x \in [0, 1]\}$ where $x =$ fraction of PoW investors

Model II: Value and Payoffs

Each technology creates value split among adopters

- Value V, Adoption $\alpha > 1$:
  - $V_W = V(xK)^{\alpha}$ and $V_S = V((1-x)K)^{\alpha}$
- Payoff functions: equal share amongst all invested units:
  - $u(W,x) = V_W \cdot (xK)^{-1} - \gamma = VK^{\alpha-1}x^{\alpha-1} - \gamma$
  - $u(S,x) = V_S \cdot ((1-x)K)^{-1} = VK^{\alpha-1}(1-x)^{\alpha-1}$
- For the purposes of this talk we restrict ourselves to the case $\alpha = 2$:
  - $u(W,x) = VKx - \gamma$
  - $u(S,x) = VK(1-x)$

An Evolutionary Game

Evolutionary game interpretation

$$P = \begin{array}{c} \\ W \\ S \end{array} \begin{array}{c} \begin{array}{cc} W & S \end{array} \\ \begin{pmatrix} VK-\gamma & -\gamma \\ 0 & VK \end{pmatrix} \end{array} \tag{G1}$$

Theorem

(G1) has three Nash equilibria: (W,W), (S,S) and one mixed. The two pure equilibria are evolutionary stable, whereas the mixed one is unstable.

Population Dynamics

Q-Learning dynamics:

$$\dot{x} = x[\underbrace{u(W,x) - \bar{u}(x)}_{\text{Replicator Dynamics}} - T \cdot \underbrace{(x \ln x + (1-x) \ln (1-x))}_{\text{Entropy}}]$$

Where $\bar{u}(x) = xu(W,x) + (1-x)u(S,x)$

Quantal Response Equilibrium (QRE): The steady states of the system, i.e., $\dot{x} = 0$.

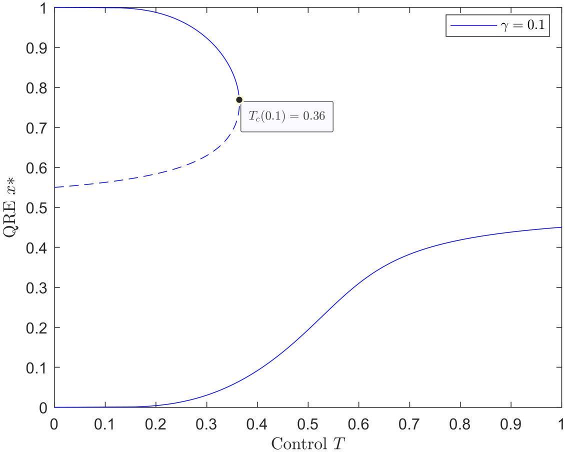
We can affect the agents' rationality by scaling the agents utilities:

$$x[\frac{u(W,x)}{c} - \frac{\bar{u}(x)}{c} - T \cdot (x \ln x + (1-x) \ln (1-x))] = 0$$

$$\iff x[u(W,x) - \bar{u}(x) - cT \cdot (x \ln x + (1-x) \ln (1-x))] = 0$$

QRE Correspondence: Visually

In our case ($\alpha = 2$): $\dot{x} = x(1-x)[2x - (1+\gamma) - T\ln\left(\frac{x}{1-x}\right)]$



QRE Correspondence: Formally

Theorem

For any $\alpha > 1$ there exists a finite sequence of temperatures $T =< T_0, T_1, \cdots >$ such as starting from an initial state x_0 and performing the following procedure for each $T_i \in T$:

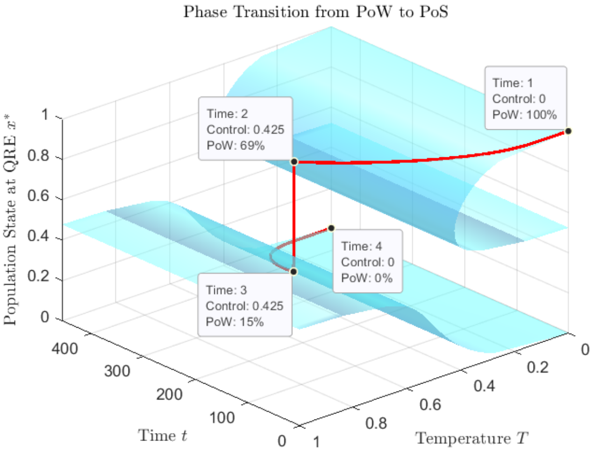
- *Scale the system's temperature at T_i, and*
- *Wait until the system converges to a QRE*

the system is going to converge to the desirable state $x = 0$ which corresponds to energy-friendly technology S.

We can reliably destabilize PoW equilibrium and converge to PoS equilibrium by introducing and removing taxes in the system.

Short Term Policy $\implies$ **Long Lasting Effects**

Conclusion


Phase Transition from PoW to PoS
Time: 1
Control: 0
PoW: 100%
Time: 2
Control: 0.425
PoW: 69%
Time: 3
Control: 0.425
PoW: 15%
Time: 4
Control: 0
PoW: 0%
Population State at QRE x^*
1
0.8
0.6
0.4
0.2
0
Time t
400
300
200
100
0
Temperature T
1
0.8
0.6
0.4
0.2
0