Bisimilarity & Simulatability of Processes Parameterized by Join Interactions

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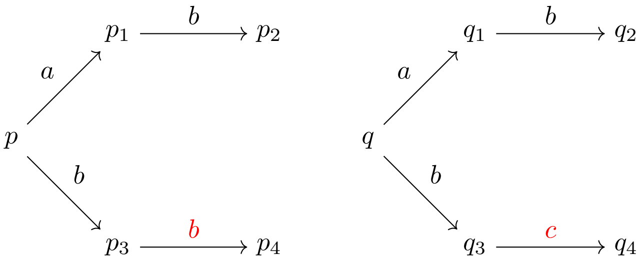
Introduction

This talk is about simulations and bisimulations.

Typical simulations and bisimulations (e.g. strong and weak ones) do not explicitly take into account the execution context.

Here we focus on behavioural relations which are parameterised by an *environment* that, intuitively, models the expected interactions of the context.

Example



p and q exhibit different behaviour.

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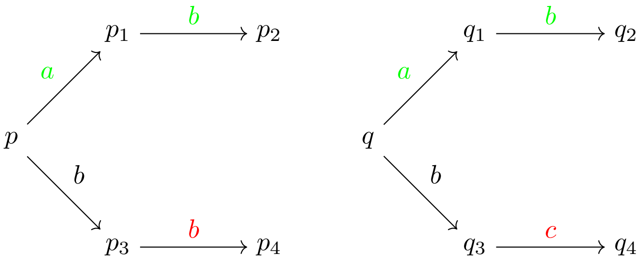

p and q exhibit different behaviour.

However, they can be considered equivalent in contexts where only the a branch is executed.

Larsen's parametrised bisimilarity

A notion of environment parametrised bisimilarity has been introduced by Kim Larsen:

(1) Kim G. Larsen. Context-Dependent Bisimulation between Processes. PhD thesis. University of Edinburgh, 1986.

(2) Kim G. Larsen. A Context Dependent Equivalence between Processes. In: TCS (1987)

Larsen's parametrised bisimilarity

How to represent the environment?

Larsen's parametrised bisimilarity

How to represent the environment?

Just as a state of an LTS...

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So we have two LTSs:

Process LTS $\mathcal{P} = \langle \mathrm{Pr}, A, \rightarrow \rangle$ and

Environment LTS $\mathcal{E} = \langle \mathrm{Env}, A, \Longrightarrow \rangle$.

Larsen's parametrised bisimilarity

An *$\mathcal{E}$-parameterized bisimulation $\mathcal{B}$* is an Env-indexed family $\mathcal{B} = \{B_f\}_{f \in \mathrm{Env}}$ of non-empty binary relations $B_f \subseteq \mathrm{Pr} \times \mathrm{Pr}$ such that:

If $p \; B_e \; q$ for $e \in \mathrm{Env}$, then if $e \overset{a}{\Longrightarrow} e'$ for $a \in A$ the following conditions hold:

(forth) $(\forall p' \in \mathrm{Pr})\big[\, p \overset{a}{\to} p' \;\Longrightarrow\; (\exists\, q' \in \mathrm{Pr})[\, q \overset{a}{\to} q' \wedge p' \; B_{e'} \; q' \,]\big]$,
(back) $(\forall q' \in \mathrm{Pr})\big[\, q \overset{a}{\to} q' \;\Longrightarrow\; (\exists\, p' \in \mathrm{Pr})[\, p \overset{a}{\to} p' \wedge p' \; B_{e'} \; q'\,]\big]$.

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We write $p \sim_e q$ if $p\ B_e\ q$ holds for some $\mathcal{E}$-parameterized bisimulation $\mathcal{B}$.

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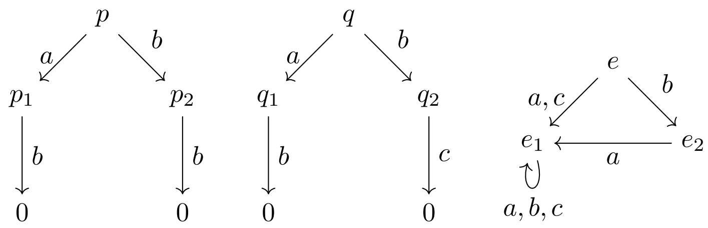

We have that $p \sim_e q$.

Notable properties of parametrised bisimilarity from Larsen's thesis (1)

Characterisation of the discrimination preorder as simulatability (a.k.a. similarity):

$$\sim_e \subseteq \sim_f \quad\Longleftrightarrow\quad f \preccurlyeq e$$

The $\Longrightarrow$ direction has been proved only for image-finite processes. The general case is still open...

Hennessy-Milner logic

Syntax:

$$\phi ::= \top \mid \neg\phi \mid \phi \wedge \phi \mid \langle a \rangle \phi$$

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Satisfaction relation:

$$p \vDash \top \;:\iff\; p \in \mathrm{Pr} \qquad p \vDash \phi_1 \wedge \phi_2 \;:\iff\; p \vDash \phi_1 \text{ and } p \vDash \phi_2$$

$$p \vDash \neg\phi_0 \;:\iff\; p \nvDash \phi_0 \qquad p \vDash \langle a \rangle \phi_0 \;:\iff\; \exists p' \in \mathrm{Pr}\,(\, p \xrightarrow{a} p' \text{ and } p' \vDash \phi_0 \,)$$

Notable properties of parametrised bisimilarity from Larsen's thesis (2)

Larsen's thesis contains a logical characterisation of $\sim_e$.

Let $\mathcal{M}(p)$ be the set of all the Hennessy-Milner formulae satisfied by p.

Let $\overline{\mathcal{L}(e)}^{\neg}$ be the negation closure of positive formulae satisfied by e.

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Then, for all image finite p, q, e:

$$p \sim_e q \iff \mathcal{M}(p) \cap \overline{{}^{\neg}\mathcal{L}(e)} = \mathcal{M}(q) \cap \overline{{}^{\neg}\mathcal{L}(e)}$$

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Compare with the logical characterisation of strong bisimilarity:

$$p \sim q \iff \mathcal{M}(p) = \mathcal{M}(q)$$

Parametrised bisimulation and (generalised pseudo-) metrics

The paper (3) shows that, under some closure properties of the environment LTS, a generalisation of parametrised bisimilarity induces a generalised pseudo-metric.

The distance between processes p and q can be defined as the largest (according to $\preccurlyeq$) environment e such that $p \sim_e q$.

(3) Ugo Dal Lago and Maurizio Murgia. Contextual Behavioural Metrics. In CONCUR 2023.

Towards ji-parametrised bisimilarity

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What if, instead, we compose the processes with the environment and then compare the resulting composite processes?

Join interaction and ji-bisimilarity

Join interaction:

$$\frac{p_1 \xrightarrow{a}_1 p_1' \qquad p_2 \xrightarrow{a}_2 p_2'}{p_1 \,\&\, p_2 \xrightarrow{a} p_1' \,\&\, p_2'}$$

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Ji-bisimilarity:

$$p \sim_{\&e} q \;:\Longleftrightarrow\; p \,\&\, e \sim q \,\&\, e$$

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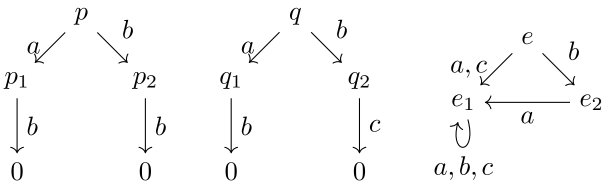

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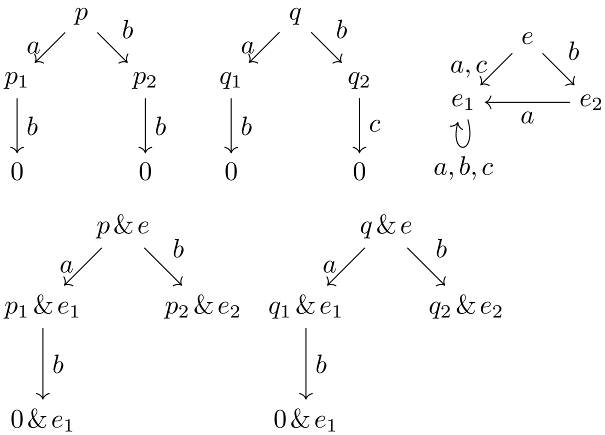

p
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c
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0
e
a, c
b
e_1
e_2
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a, b, c
$p \,\&\, e$
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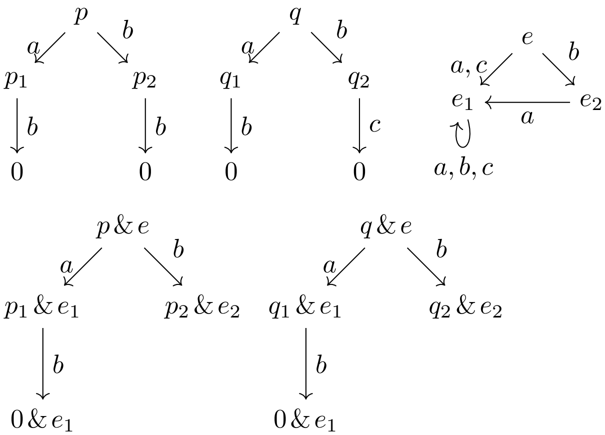

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On deterministic environments, ji-bisimilarity implies parametrised bisimilarity, again by a simple coinductive argument.

In general, ji-bisimilarity is strictly larger than parametrised bisimilarity: see next slide.

Parameterized bisimilarity $\neq$ ji-parameterized bisimilarity

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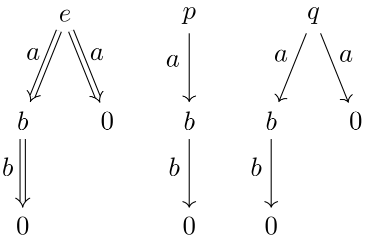

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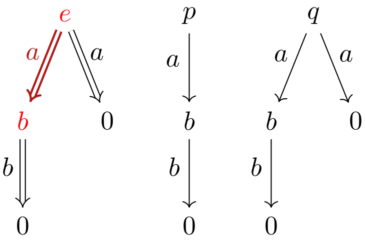

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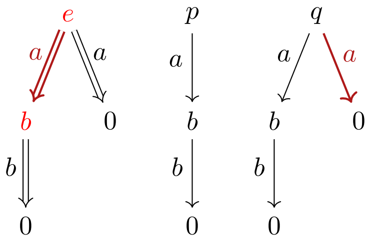

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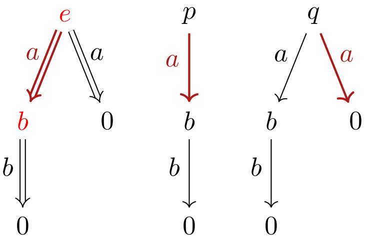

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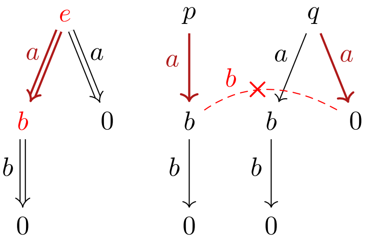

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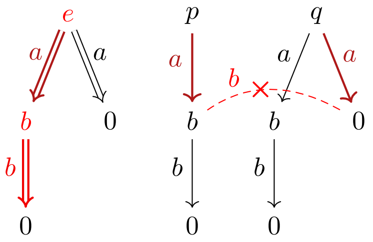

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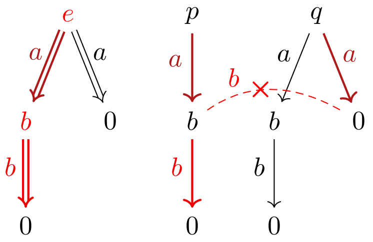

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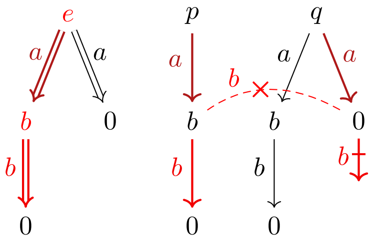

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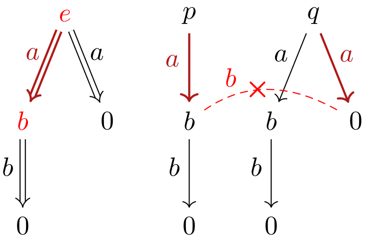

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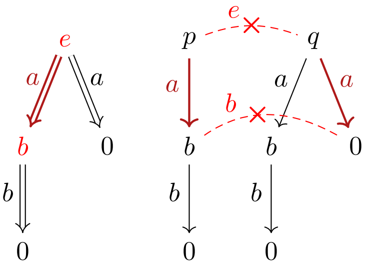

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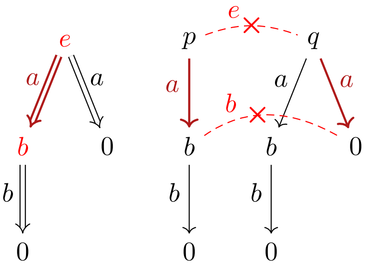

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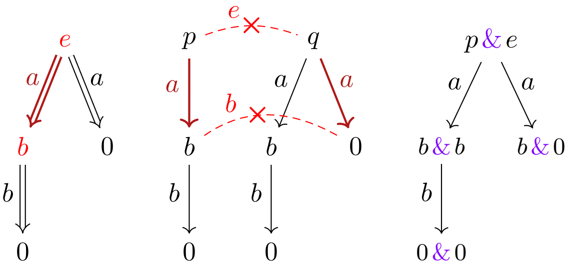

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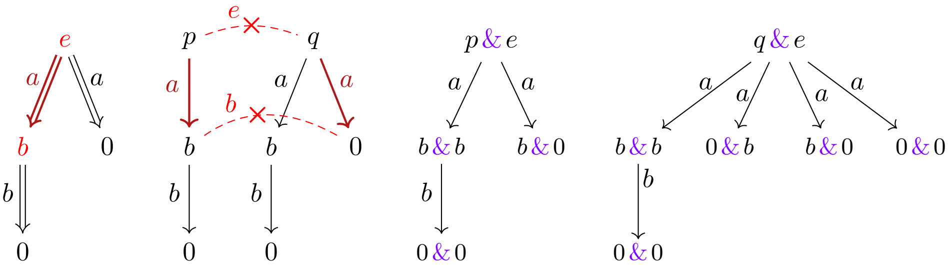

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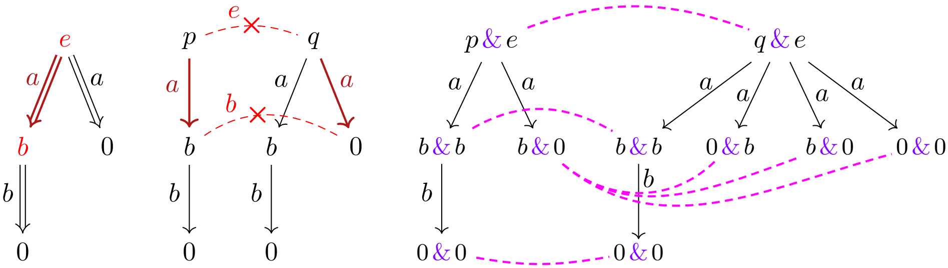

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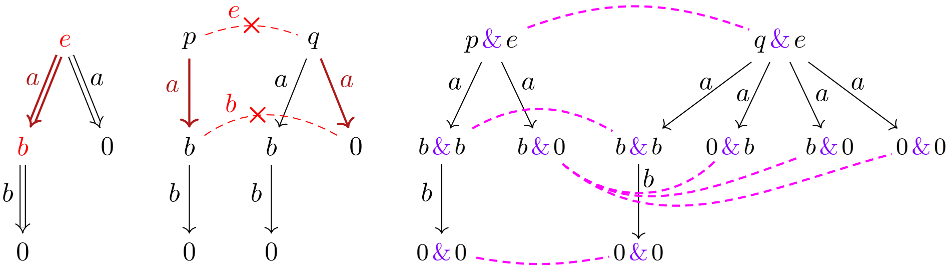

$p \not\sim_e q \qquad\qquad p \sim_{\&e} q$

Making parametrised bisimilarity and ji-bisimilarity equivalent

Right-determinizing join-interaction:

$$\frac{p \xrightarrow{a} p' \qquad e \overset{a}{\Longrightarrow} e'}{p \,\&_{\bullet}\, e \xrightarrow{\langle a, e' \rangle} p' \,\&_{\bullet}\, e'}$$

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It holds that:

$$p \mathbin{\&_\bullet} e \sim q \mathbin{\&_\bullet} e \iff p \sim_e q$$

Parameterized bisimilarity $\hat{=}$ (ji)$^\bullet$-parameterized bisimilarity

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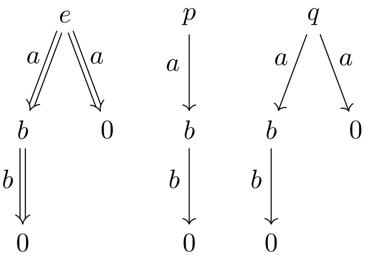

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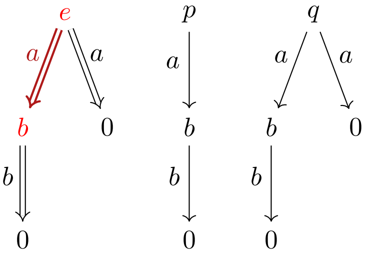

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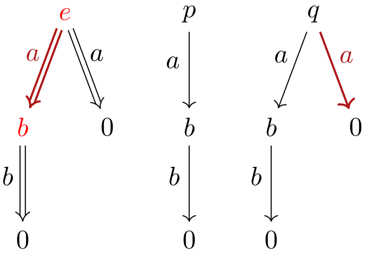

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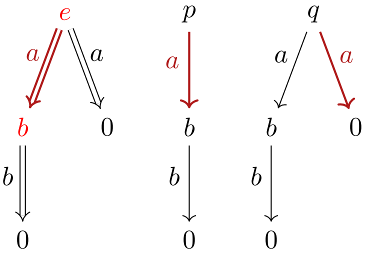

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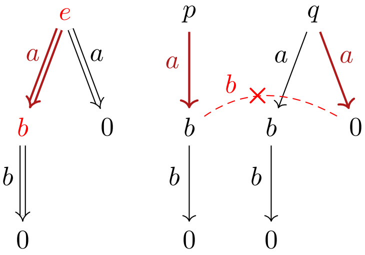

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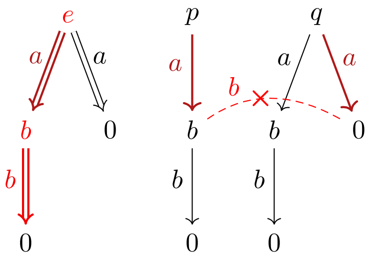

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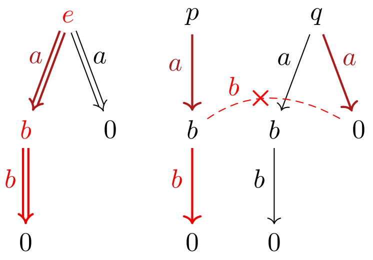

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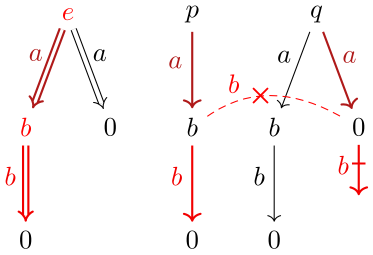

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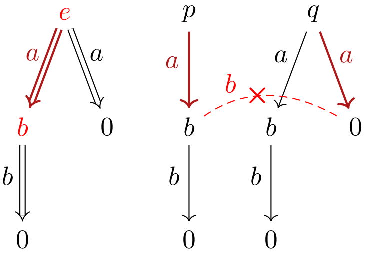

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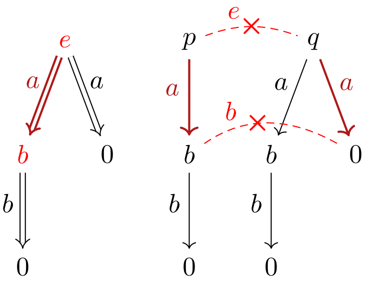

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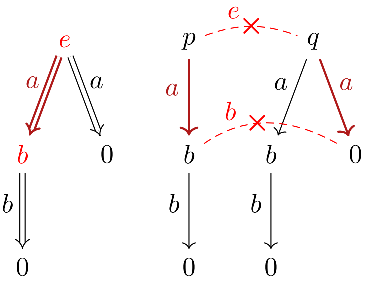

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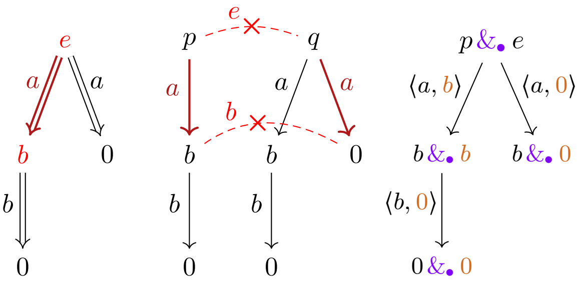

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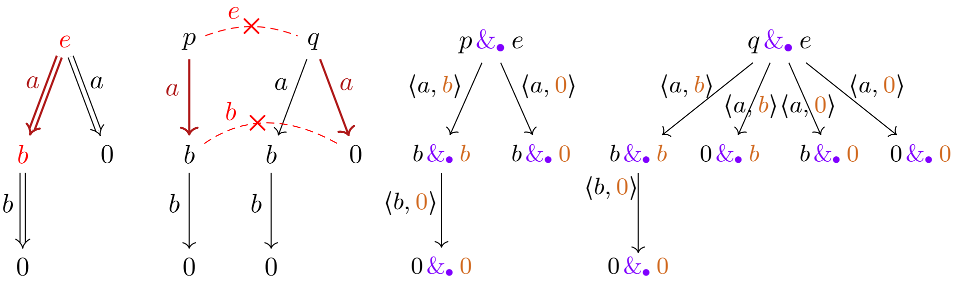

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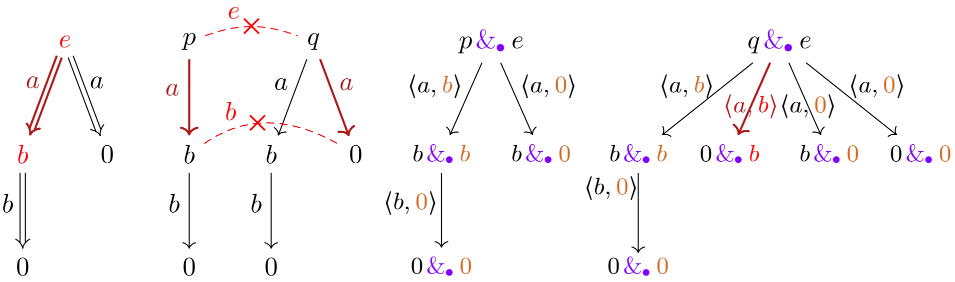

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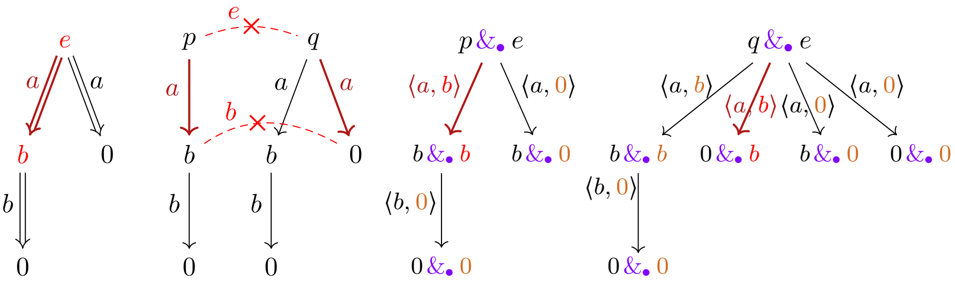

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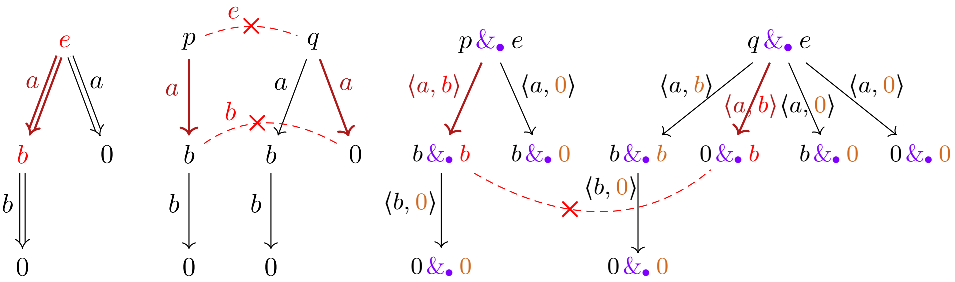

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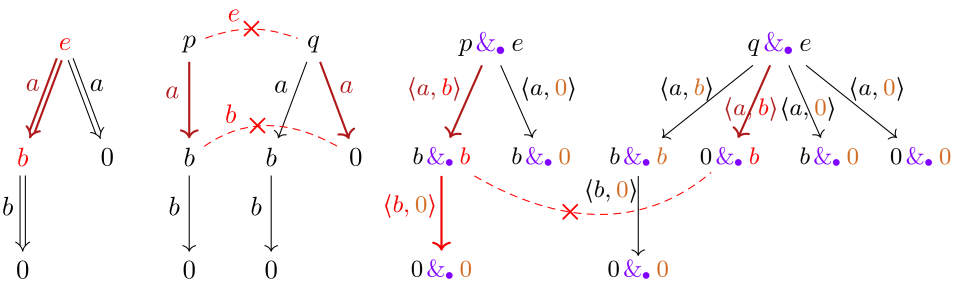

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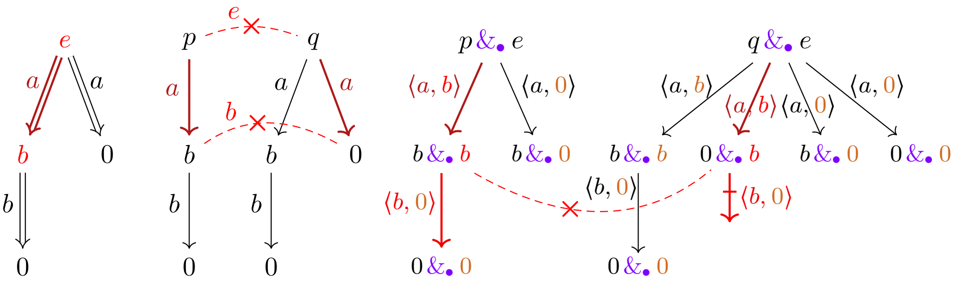

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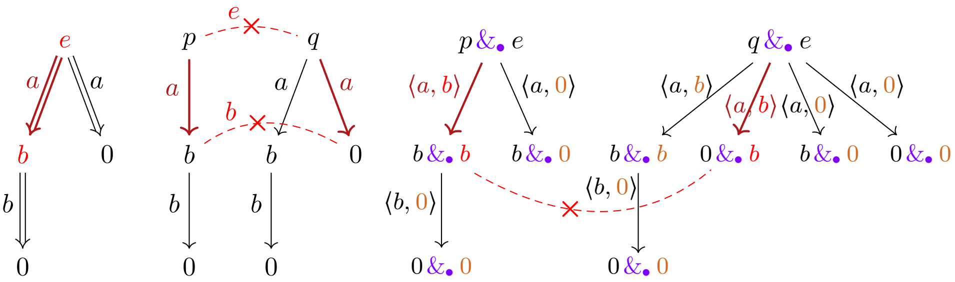

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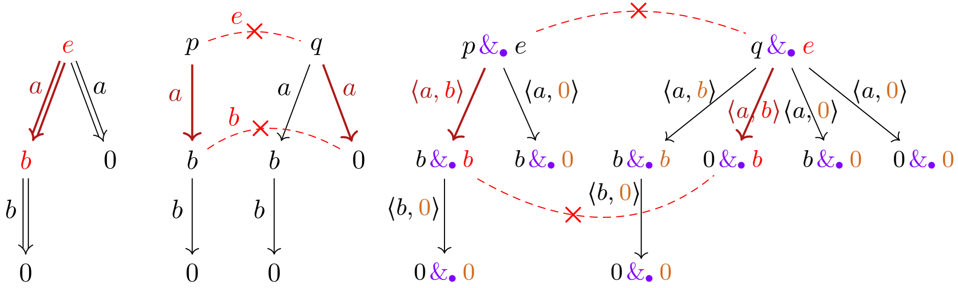

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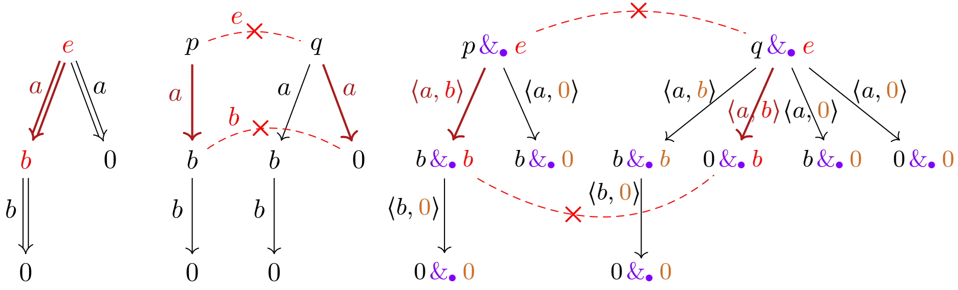

$p \not\sim_e q$ $\qquad$ $p \not\sim_{\&_\bullet e} q$

Simulation

Parametrised simulation:

If $p \, B_e \, q$ for $e \in \mathrm{Env}$, then if $e \overset{a}{\Longrightarrow} e'$ for $a \in A$ the following condition holds:

$$\text{(forth)} \;\; (\forall p' \in \mathrm{Pr})\big[\, p \xrightarrow{a} p' \implies (\exists\, q' \in \mathrm{Pr})[\, q \xrightarrow{a} q' \wedge p' \, B_{e'} \, q' \,]\big] \, .$$

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Ji-simulatability:

$$p \preccurlyeq_{\&e} q :\Longleftrightarrow\ p \,\&\, e \preccurlyeq q \,\&\, e$$

On parametrised simulatability vs ji-simulatability

Parametrised simulatability implies ji-simulatability, by a simple coinductive argument.

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Interestingly, the reverse implication holds too: ji-simulatability implies parametrised simulatability.

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Parametrised simulatability implies ji-simulatability, by a simple coinductive argument.

Interestingly, the reverse implication holds too: ji-simulatability implies parametrised simulatability.

We remark that the latter implication does not hold for bisimilarity.

On ji-simulatability implies parametrised simulatability

A crucial stepping stone in establishing that ji-simulatability implies parametrised simulatability is:

If, for some f, $p \mathbin{\&} e \preccurlyeq q \mathbin{\&} f$ then $p \preccurlyeq_e q$

On ji-simulatability implies parametrised simulatability

A crucial stepping stone in establishing that ji-simulatability implies parametrised simulatability is:

$$\text{If, for some } f,\ p \,\&\, e \precsim q \,\&\, f \text{ then } p \precsim_e q$$

The thesis follows from the above.

Logical characterisation of ji-simulatability

Let $\mathcal{L}(p)$ be the set of positive formulae (that is, formulae where negations do not occur) satisfied by p.

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We established that ji$^{\bullet}$-bisimilarity and parametrised bisimilarity coincide.

We established that ji-simulatability and parametrised simulatability coincide.

We provided a logical characterisation of ji-simulatability (and hence of parametrised simulatability).

Future works and open problems

Logical characterisation of $\sim_{\&e}$.

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Logical characterisation of $\sim_{\&e}$.

Characterisation of the discrimination preorder of $\sim_{\&e}$.

Thank you for your attention!